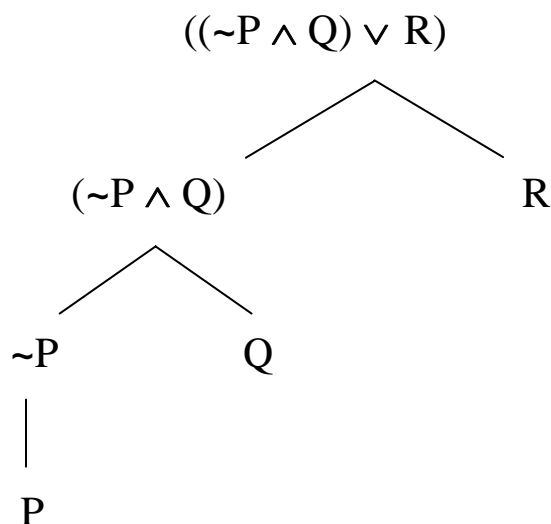


Scope and the Main Connective

We remarked earlier that without an understanding of sentence construction, our translation skills would be swamped by any but the simplest cases. Here we make good on that claim, noting how some simple points of formal construction lend focus to the contours of complex English sentences.

Recall that the molecule-building symbols tilde, wedge, and vel are called **connectives**. The **last** connective added in the process of building a formal sentence is the **main connective** of that sentence.

So while the following sentence has three different connectives, the vel is the *main* connective of the whole sentence. That's why the sentence as a whole is a *disjunction* – even though it also contains a wedge and a tilde.



The smaller sentence(s) which a connective attaches to are called the **scope** of that connective.¹ (Since the wedge and vel come between *two* sentences, a wedge or vel has a split, two-part scope.) For each molecular sentence the construction tree lists, directly below that sentence, the scope of its main connective. So in the above tree, the scope of the tilde is “P”; the (two-part) scope of the wedge is “~P” and “Q”; and the (two-part) scope of the vel is “(~P ∧ Q)” and “R”.

¹ Following the usage of, e.g., Kleene 1967/2002: 8

1. Turf Battles in English and Formalese. The scope of a connective is, in effect, the formal real estate which that connective controls; and the more real estate a connective controls, the more power it has in determining the nature of the complete formal sentence. That's why the last connective added in construction – the *main connective* – determines what sort of sentence the final result is: since the main connective has the widest scope, it has dominion over everything else in that sentence.

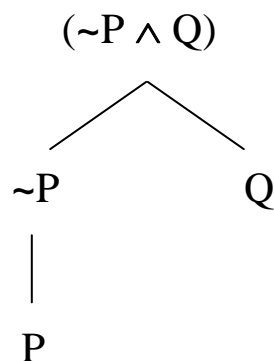
So in formal sentences with more than one connective we can picture each of those connectives competing to be the 'king of the hill,' the main connective. But as in other competitions, there can be only one winner: in the struggle over the formal real estate, one connective will take the gold medal and be crowned main connective, while its competitors will take only a silver or bronze and be relegated to some smaller part of the sentence.

For instance, both of the following sentences contain a tilde and wedge – each connective striving to be the main connective of the whole sentence.

(1) $(\sim P \wedge Q)$

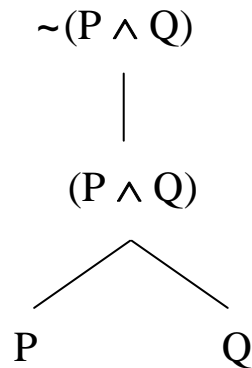
(2) $\sim(P \wedge Q)$

But the competition ends differently in the two cases. The first sentence is constructed like so.



Here the *wedge* is the last connective added – the main connective – making the whole sentence a *conjunction*. The tilde is shunted into the left suburbs of the sentence, and must content itself with ruling over its smaller scope sentence, “P”.

The second sentence is constructed like so.



Here the *tilde* wins the competition to be main connective, making the sentence as a whole a *negation*. The tilde has the larger scope, “ $(P \wedge Q)$,” while the wedge must settle for the smaller (two-part) scope “ P ” and “ Q ”.

In such scope competition we say that the connective with the larger scope has “**wide scope**” (compared to its competitor), while the connective with the smaller scope has “**narrow scope**” (compared to its competitor). So in “ $(\sim P \wedge Q)$ ” the wedge has wide scope, while the tilde has narrow scope; whereas in “ $\sim(P \wedge Q)$ ” the tilde takes wide scope, and the wedge has narrow scope.

Competition over scope might seem a purely technical issue of Formalese. Yet it arises equally in English sentences, in ways that affect matters of truth and validity. For though English lacks *connectives*, it possesses a variety of now-familiar *form phrases*. And as these phrases battle over turf the same way connectives do in Formalese, distinctions of scope are mirrored in English.

The difference between the following two English sentences is obvious enough.

- (3) We’re not having ice cream, but we’re having cake.
- (4) We’re not having *both* ice cream *and* cake.

Sentence (3) states exactly how dessert will go: cake without ice cream. Sentence (4) is far less specific, ruling out only one dessert option (cake with ice cream) while leaving several open. For all (4) tells us, we might have (i) cake without ice cream, (ii) ice cream without cake, or (iii) neither.

Any English speaker can see that difference intuitively. But note that Sentences (3) and (4) translate into Formalese sentences (1) and (2).

P: We're having ice cream

Q: We're having cake.

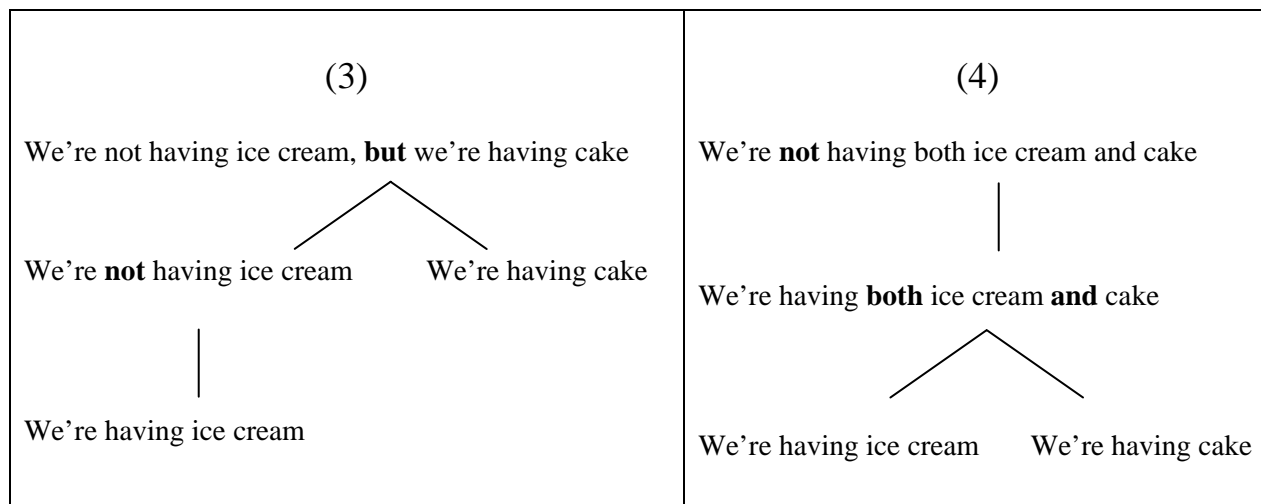
(3) We're not having ice cream, but we're having cake. (1) $(\sim P \wedge Q)$

(4) We're not having *both* ice cream *and* cake. (2) $\sim(P \wedge Q)$

Thanks to formal translation we see that the difference between English sentences (3) and (4) amounts to a difference of scope – though now in terms of competing English form phrases rather than formal connectives.

In (3) the *main* form phrase of the sentence is “but,” gluing together the smaller sentences “*We're not having ice cream*” and “*We're having cake*”. As the main form phrase of the sentence, “but” makes Sentence (3) a **conjunction**; and the two smaller sentences being glued together form the two-part scope of the conjunction phrase “but”.

Sentence (4) is a denial – an English language **negation**. What it's a denial *of* is the smaller claim “*We're having both ice cream and cake*”. As a negation, the main form phrase is “not”; and the smaller conjunction being denied is the scope of that negation phrase.



Simple English examples show how this difference can affect validity.

Argument A	Argument B
<p>We're having neither ice cream nor cake</p> <hr/> <p>∴ (3) We're not having ice cream, but we're having cake.</p>	<p>We're having neither ice cream nor cake</p> <hr/> <p>∴ (4) We're not having <i>both</i> ice cream <i>and</i> cake.</p>

Argument (B) is intuitively **valid**: if it's true we're having neither ice cream nor cake, it's certainly true that we aren't having both.

Not so with Argument (A): in a situation where we have neither ice cream nor cake, it must be *false* that we're having ice-cream-less cake. Since the premise can be true while the conclusion is false, Argument (A) is **invalid**.

Of course the conclusion of Argument (A) is just Sentence (3) from above, while the conclusion of (B) is Sentence (4). So the simple matter of which form phrase acts as 'main connective' – the conjunction phrase in (3), the negation phrase in (4) – makes a world of difference to validity. And that means getting clear on the main connective in formal translation will be essential for correctly evaluating the validity of English arguments.

Incidentally, this illustrates as well the importance of the humble parenthesis in Formalese. Having recognized the difference between Sentences (1) and (2), we should note that the sole bit of formality distinguishing them is the arrangement of their parentheses: in Sentence (1) the tilde is tucked inside the left parenthesis, while in (2) the tilde is outside.

- (1) ($\sim P \wedge Q$)
- (2) $\sim(P \wedge Q)$

So parentheses are indispensable for making clear issues of scope and main connective.

Missing parentheses make the following a bogus piece of Formalese gibberish.

$$(5) \sim P \wedge Q$$

Not knowing whether the tilde applies to everything that follows or just to “P,” we can’t tell whether we’re looking at a negation or a conjunction. And for reasons just rehearsed, that can make all the difference in the world to validity.

2. Scope and the Main Connective: English Clues. Appreciating the importance of correctly identifying the main connective, we list some English clues to this end.

First, certain bits of English serve the same essential function as the *left parenthesis* does in Formalese. While the “**either**” in “either... or,” and “**both**” in “both... and” are optional, their placement can tip the scales when an English conjunction or disjunction is dueling with a negation phrase for wider scope. So in Sentence (6), “either” wraps outside “not,” just as the left parenthesis outflanks the tilde in formal Sentence (7).

$$(6) [\textbf{Either} \text{ we're } \underline{\text{not}} \text{ having ice cream, or we're having cake}] \quad (7) (\sim P \vee Q)$$

In Sentence (8), “not” attaches to the left of the “either,” just as the tilde does with the left parenthesis in (9) – rendering both negations.

$$(8) \underline{\text{It's not the case that}} \text{ we're having } [\textbf{either} \text{ ice cream or cake}] \quad (9) \sim(P \vee Q)$$

Likewise in Sentence (4) we negated an English conjunction by first building a conjunction with “both,” then attaching “not” outside the “both”.

$$(4) \text{ We're } \underline{\text{not}} \text{ having } [\textbf{both} \text{ ice cream and cake}] \quad (2) \sim(P \wedge Q)$$

Second, in a duel for dominance between two-place form phrases – e.g., a conjunction and a disjunction phrase – the **comma** is often the crucial English clue. For the main form phrase of the sentence, acting as the main connective, marks the biggest *break* in the sentence (the gap coming right between the two scope sentences being glued together). And the comma is a natural English means of marking that main break.

Note that the following two sentences are word-for-word identical, differing only in where the comma falls.

(10) Either we'll have ice cream, **or** we'll have cake and we'll have champagne

(11) Either we'll have ice cream or we'll have cake, **and** we'll have champagne

Sentences (10) and (11) make very different promises – for example, champagne is a sure thing with Sentence (11), but not so with (10). The placement of a single comma makes all the difference: in (10) the comma falls right beside “or,” marking that as the main form phrase and making (10) a *disjunction*.

(10) Either we'll have ice cream, **or** we'll have cake and we'll have champagne

The left half of this disjunction is the subject matter sentence “*We'll have ice cream,*” while the right half is the conjunction “*We'll have cake and we'll have champagne*”. Sentence (10) is thus translated as formal sentence (12)

P: We'll have ice cream

Q: We'll have cake

R: We'll have champagne

(10) Either we'll have ice cream, **or** we'll have cake and we'll have champagne

(12) $(P \underline{\vee} (Q \wedge R))$

In Sentence (11) the comma falls instead beside “and” – rendering that the main form phrase, and making (11) a *conjunction*. Its left part is the disjunction “*Either we'll have ice cream or we'll have cake,*” while the right is “*We'll have champagne*”. Using the same translation table, Sentence (11) translates as (13).

(11) Either we'll have ice cream or we'll have cake, **and** we'll have champagne

(13) $((P \vee Q) \underline{\wedge} R)$

It must be stressed that the complexities of English stand in the way of a full-proof set of techniques for English-to-Formalese translation. Still, the small toolbox of clues we've assembled so far will prove adequate for a wide array of cases.